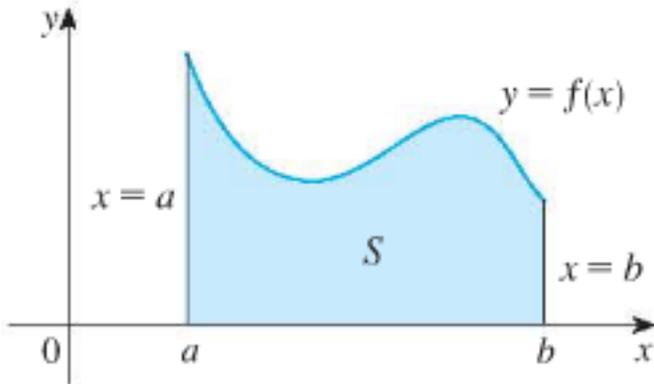


Recall: what does f' tell us about the function f ?

The **antiderivative** or **integral** of f represents _____.

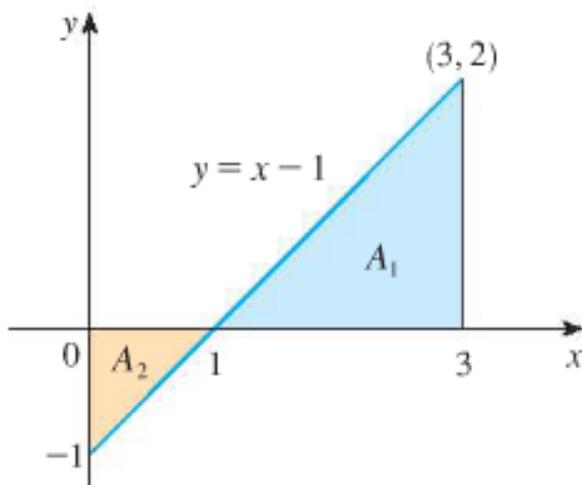
The “Problem Child”



It's a challenge to determine the area under “non-standard” shapes such as the bounded area under a function curve.

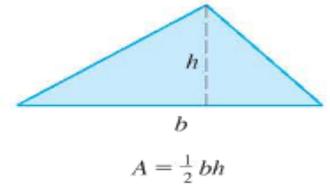
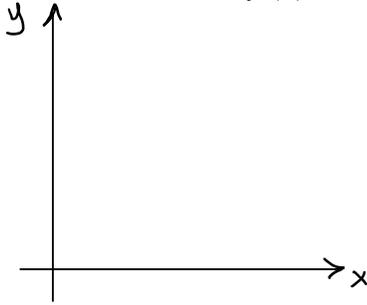
If the area under curve is a STANDARD shape then known area formulas can be used.

Caveat: If area lies **BELOW** x -axis, that region is **negative** (subtracted).

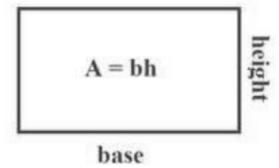
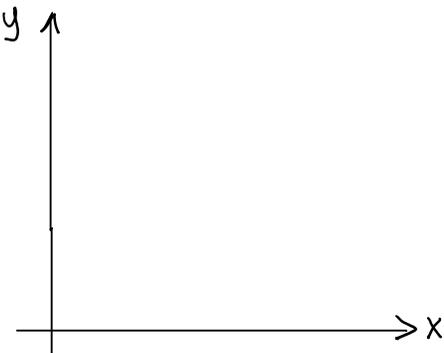


Total area of shaded regions:

ex. Find the area under $f(x) = x$ from $x = 0$ to $x = 4$.



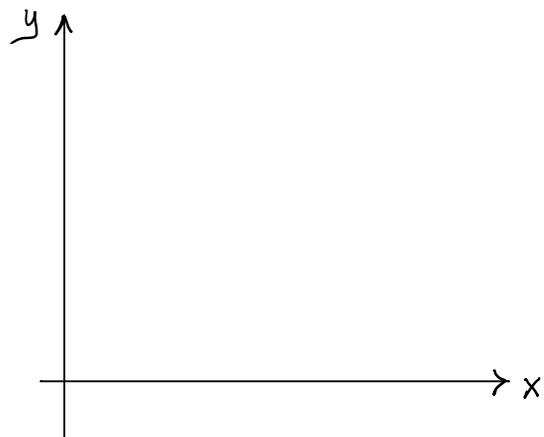
ex. Find the area under $f(x) = x + 1$ from $x = 0$ to $x = 4$.



DO: Find the area under $f(x) = 4$ from $x = 0$ to $x = 4$.

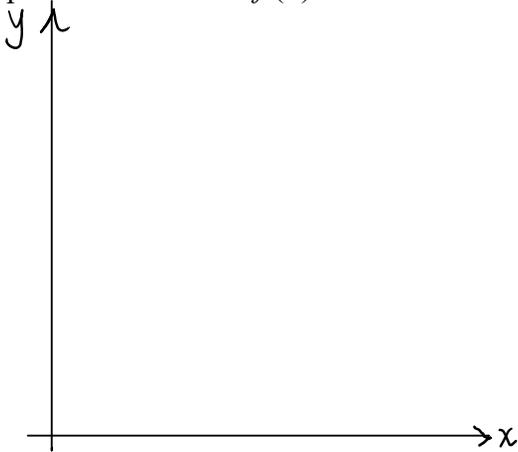


Now, let's look at it differently:



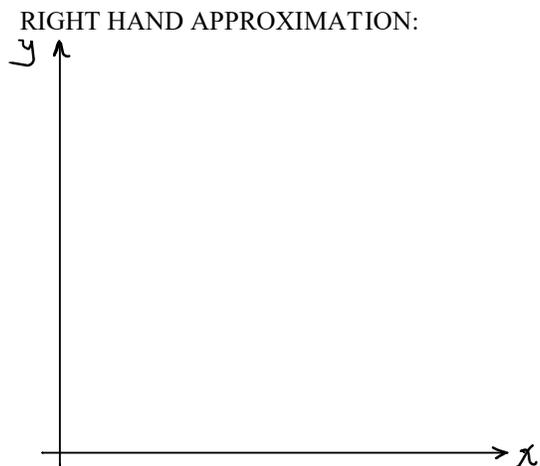
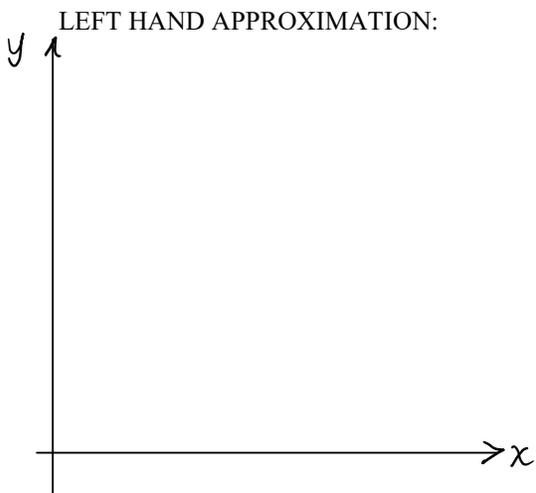
Now, let's apply the rectangle method to **ESTIMATE** the area under parabolas.

ex. Split the area under $f(x) = x^2$ from $x = 1$ to $x = 4$ into 3 regions.



For now, use rectangles to **ESTIMATE** the exact area under curve:

ex. **ESTIMATE** the area under $f(x) = x^2$ from $x = 1$ to $x = 4$ using 3 approximating rectangles.



Computation Efficiency Tip: factor out base

Approximating the area under a curve
using the sum of areas of rectangles
is called **Riemann Sum**.

Note: bases of rectangles must be the SAME

ex. ESTIMATE the area under $f(x) = x^2$ from $x = 1$ to $x = 4$ using **6** approximating rectangles.

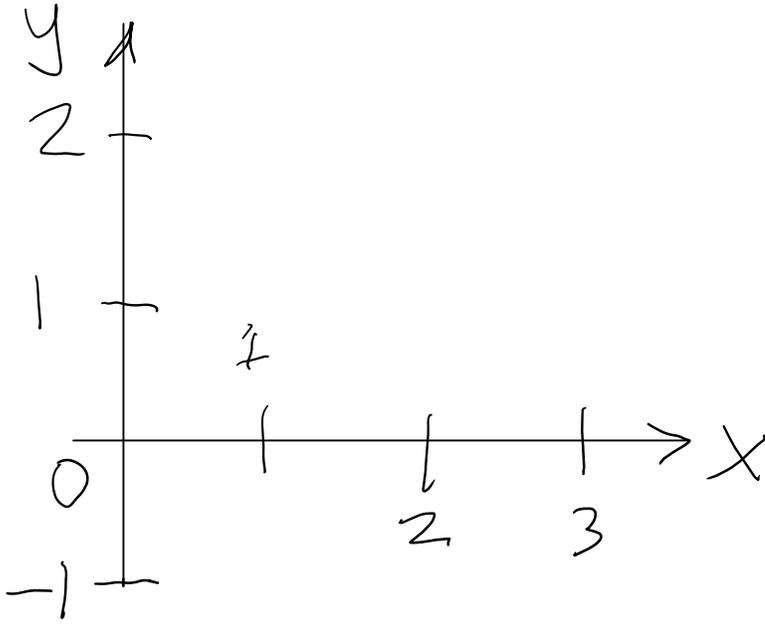
NOTE: using more boxes increases the accuracy of the estimate

Now, let's determine x -values differently:

ex. SET UP the area under $f(x) = x^2$ from $x = 1$ to $x = 4$ using **5** *LEFT-HAND* approximating rectangles.

ex. SET UP the area under $f(x) = x^2$ from $x = 1$ to $x = 4$ using **5** *RIGHT-HAND* approximating rectangles.

Revisit: EXACT area under $f(x) = x - 1$ from $x = 0$ to $x = 3$.



ex. SET UP the area under $f(x) = \sin x$ from $x = 0$ to $x = \frac{3\pi}{2}$ using **6 LEFT-HAND** approximating rectangles.

